June 2010
Further Pure Mathematics FP3 6669
Mark Scheme

Question Number	Scheme	Marks
1.	$\pm\dfrac{a}{e}=8, \quad \pm ae=2$	B1, B1
	$\dfrac{a}{e}\times ae=a^2=16$	
	$a=4$	B1
	$b^2=a^2(1-e^2)=a^2-a^2e^2$	
	$\Rightarrow b^2=16-4=12$	M1
	$\Rightarrow b=\sqrt{12}=2\sqrt{3}$	A1 (5)
		5

Question Number	Scheme	Marks
2.	$x^2+4x+13=(x+2)^2+9$	B1
	$\int \frac{1}{(x+2)^2+9}dx=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$	M1 A1
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}\left(\arctan 1-\arctan 0\right)$	M1
	$=\frac{\pi}{12}$	A1 (5)
		5

Question Number	Scheme	Marks
3(a)	$rhs = 1 + 2\sinh^2 x = 1 + 2\left(\dfrac{e^x - e^{-x}}{2}\right)^2$	M1
	$= \dfrac{2 + e^{2x} - 2 + e^{-2x}}{2}$	M1
	$= \dfrac{e^{2x} + e^{-2x}}{2} = \cosh 2x = lhs$ *	A1 (3)
(b)	$1 + 2\sinh^2 x - 3\sinh x = 15$	
	$2\sinh^2 x - 3\sinh x - 14 = 0$	M1
	$(\sinh x + 2)(2\sinh x - 7) = 0$	M1
	$\sinh x = -2, \dfrac{7}{2}$	A1
	$x = \ln\left(-2 + \sqrt{(-2)^2 + 1)}\right) = \ln\left(-2 + \sqrt{5}\right)$	M1
	$x = \ln\left(\dfrac{7}{2} + \sqrt{\left(\dfrac{7}{2}\right)^2 + 1)}\right) = \ln\left(\dfrac{7 + \sqrt{53}}{2}\right)$	A1 (5)
		8

Question Number	Scheme	Marks
4(a)	$\int (a-x)^n \cos x\,dx = (a-x)^n \sin x + \int n(a-x)^{n-1} \sin x\,dx$	M1A1
	$\left[(a-x)^n \sin x\right]_0^a = 0$	A1
	$= -n(a-x)^{n-1}\cos x - \int n(n-1)(a-x)^{n-2} \cos x\,dx$	dM1
	$I_n = na^{n-1} - n(n-1)I_{n-2}$ *	A1 (5)
(b)	$I_2 = 2\left(\frac{\pi}{2}\right) - 2\int_0^{\frac{\pi}{2}} \cos x\,dx$	M1 A1
	$= \pi - 2\left[\sin x\right]_0^{\frac{\pi}{2}} = \pi - 2$	A1 (3)
		8

Question Number	Scheme	Marks
5(a)	$\frac{dy}{dx} = 2\text{ar}\cosh(3x) \times \frac{3}{\sqrt{9x^2-1}}$	M1A1A1
	$\sqrt{9x^2-1}\frac{dy}{dx} = 6\text{ar}\cosh(3x)$	
	$(9x^2-1)\left(\frac{dy}{dx}\right)^2 = 36\left(\text{ar}\cosh(3x)\right)^2$	dM1
	$(9x^2-1)\left(\frac{dy}{dx}\right)^2 = 36y$ *	A1 (5)
(b)	$\left\{18x\left(\frac{dy}{dx}\right)^2 + (9x^2-1)\times 2\frac{dy}{dx}\times\frac{d^2y}{dx^2}\right\} = 36\frac{dy}{dx}$	M1 {A1} A1
	$(9x^2-1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$ *	A1 (4)
		9

Question Number	Scheme	Marks
6(a)	$\begin{pmatrix}1 & 0 & 3\\ 0 & -2 & 1\\ k & 0 & 1\end{pmatrix}\begin{pmatrix}6\\1\\6\end{pmatrix}=\lambda\begin{pmatrix}6\\1\\6\end{pmatrix}$ $\begin{pmatrix}24\\4\\6k+6\end{pmatrix}=\begin{pmatrix}6\lambda\\ \lambda\\ 6\lambda\end{pmatrix}$ Uses the first or second row to obtain $\lambda = 4$	M1A1 (2)
(b)	Uses the third row and their $\lambda = 4$ to obtain $6k+6=24 \Rightarrow k=3$ *	M1 A1 (2)
(c)	$\begin{vmatrix}1-\lambda & 0 & 3\\ 0 & -2-\lambda & 1\\ 3 & 0 & 1-\lambda\end{vmatrix}=0$ $\Rightarrow (1-\lambda)\big((-2-\lambda)(1-\lambda)-0\big)-0\big(0(1-\lambda)-3\big)+3\big(0-3(-2-\lambda)\big)=0$ $\Rightarrow (1-\lambda)(-2-\lambda)(1-\lambda)+9(2+\lambda)=(2+\lambda)\left(9-(1-\lambda)^2\right)=0$ $\left(\lambda^3-12\lambda-16=0\right)$ $\Rightarrow (\lambda+2)\left(\lambda^2-2\lambda-8\right)=0$ $\Rightarrow (\lambda+2)(\lambda+2)(\lambda-4)=0$ $\lambda=-2, 4$	M1 A1 M1 A1 (4)
(d)	Parametric form of l_1 : $(t+2, -3t, 4t-1)$ $\begin{pmatrix}1 & 0 & 3\\ 0 & -2 & 1\\ 3 & 0 & 1\end{pmatrix}\begin{pmatrix}t+2\\ -3t\\ 4t-1\end{pmatrix}=\begin{pmatrix}13t-1\\ 10t-1\\ 7t+5\end{pmatrix}$ Cartesian equations of l_2 : $\dfrac{x+1}{13}=\dfrac{y+1}{10}=\dfrac{z-5}{7}$	M1 M1 A1 ddM1A1(5) **13**

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	M1 A2(1,0)
	$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 5$	
	$\mathbf{r} \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1A1 (5)
(b)	Equation of l is $\mathbf{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	M1
	At intersection $\begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1
	$\Rightarrow 6+t+4(13+4t)+2(5+2t)=5 \Rightarrow t=-3$	M1
	N is $(3,1,-1)$ *	A1 (4)
(c)	$\overrightarrow{PN} \bullet \overrightarrow{PR} = (-3\mathbf{i}-12\mathbf{j}-6\mathbf{k}) \bullet (-5\mathbf{i}-13\mathbf{j}-3\mathbf{k}) = 189$	M1 A1ft
	$\sqrt{9+144+36}\sqrt{25+169+9}\cos NPR = 189$	A1
	$NX = NP\sin NPR = \sqrt{189}\sin NPR = 3.61$	M1A1 (5) **14**

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = 4\sec t\tan t \quad \frac{dy}{dt} = 2\sec^2 t$	B1 (both)
	$\frac{dy}{dx} = \frac{2\sec^2 t}{4\sec t\tan t} \quad \left(= \frac{1}{2\sin t}\right)$	M1
	$y - 2\tan t = \frac{1}{2\sin t}(x - 4\sec t)$	M1 A1
	$2y\sin t - \frac{4\sin^2 t}{\cos t} = x - \frac{4}{\cos t}$	
	$2y\sin t = x - \frac{4 - 4\sin^2 t}{\cos t} = x - 4\cos t$ *	A1 (5)
(b)	Gradient of l_2 is $-2\sin t$	M1
	$y = -2x\sin t$ (2)	A1
	$2(-2x\sin t)\sin t = x - 4\cos t \Rightarrow x = \frac{4\cos t}{1 + 4\sin^2 t}$ (1)	M1 A1
	$y = \frac{-8\sin t\cos t}{1 + 4\sin^2 t}$	M1 A1
	$(x^2 + y^2)^2 = \left(\frac{16\cos^2 t}{(1+4\sin^2 t)^2} + \frac{64\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2}\right)^2$	
	$= \frac{256\cos^4 t}{(1+4\sin^2 t)^4}(1+4\sin^2 t)^2 = \frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	M1
	$16x^2 - 4y^2 = \frac{256\cos^2 t}{(1+4\sin^2 t)^2} - \frac{256\sin^2 t\cos^2 t}{(1+4\sin^2 t)^2} = \frac{256\cos^4 t}{(1+4\sin^2 t)^2}$	A1 (8) **13**